Reasoning Algebraically

in Upper Elementary

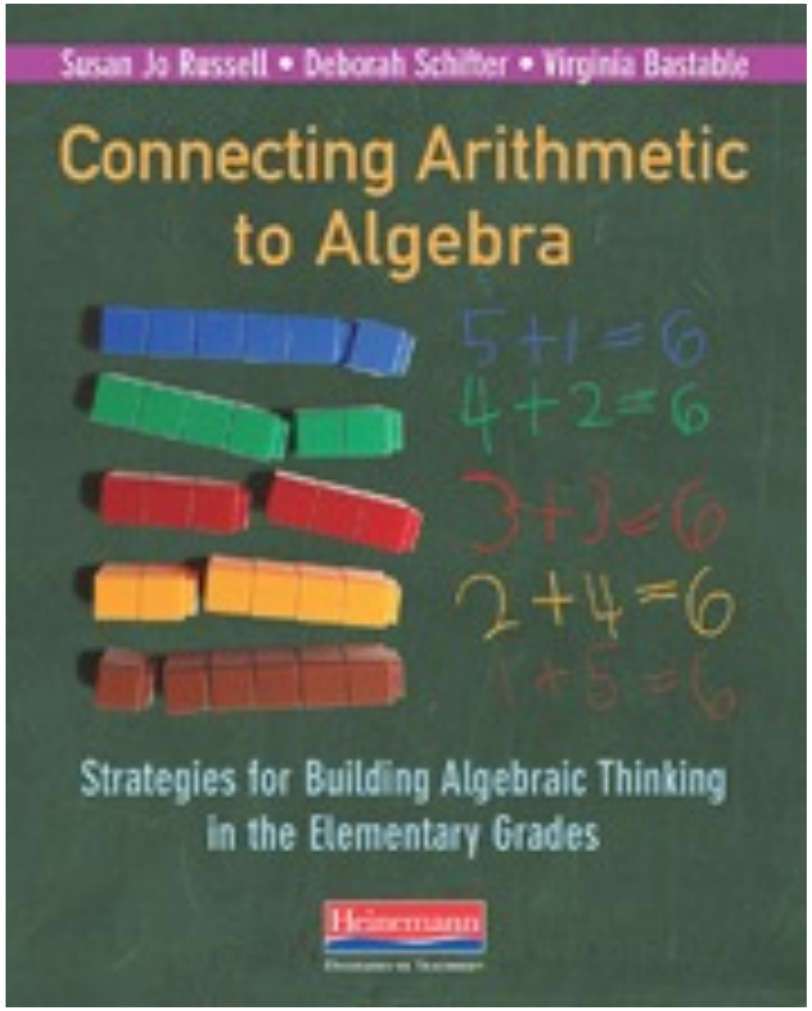
THE MONTESSORI EVENT March 16-19, 2023 Boston & Online

Presented by:

Lena Harwood Pacheco
Elementary Guide (I-II)
MA in Mathematics Teaching

Reasoning Algebraically

in Upper Elementary



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Learning Objectives

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Participants will be able to identify the **five steps** of **algebraic inquiry.**

Learning Objectives

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- Identify **algebraic concepts** in the UE curriculum
- Discover **which materials best illuminate** these concepts
- Develop best practices for **facilitating inquiry**

Reasoning Algebraically

- Math Task
- Analyze the Task
- 5 Steps
- Lesson Planning

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Math Task

- Use materials
- Look for patterns
- Will the pattern always hold true?

Reasoning Algebraically

Math Task: Integers

PART ONE

Directions: For each of the following sets of equations, solve the equation by creating a model using Montessori math materials.

Helpful Hints:
- Consider what each factor represents in a multiplication equation.
- Consider what multiplication means and how to represent that.
- Pay close attention to how the model you create needs to shift or change for each of the equations in the set.

Set 1	Set 2
3 • 4	2 • 5
3 • -4	2 • -5
-3 • 4	-2 • 5
-3 • -4	-2 • -5

PART TWO:

1. What do you notice about your answers when you multiply a negative integer by a negative integer?
2. Do you think that will always be true? Generate more equations to test your conjecture.
3. How could you adjust your model or representation so that you could prove this would always be true regardless of the numbers?

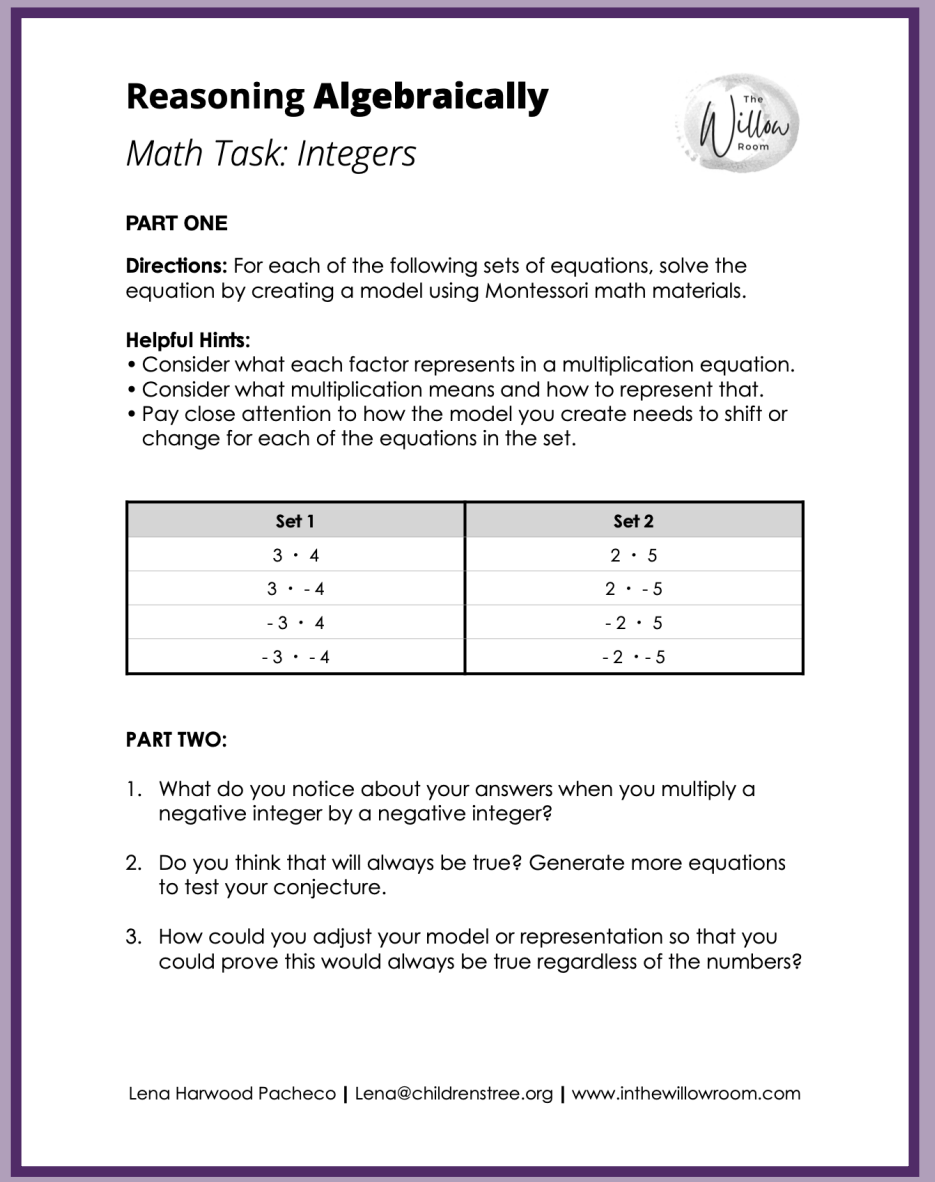
Lena Harwood Pacheco | Lena@childrenstree.org | www.inthewillowroom.com

Handout 1

Math Task

Caveat:

There is no need to pretend.



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Handout 1

Math Task

- Use materials
- Look for patterns
- Will the pattern always hold true?

10 minutes

Reasoning Algebraically

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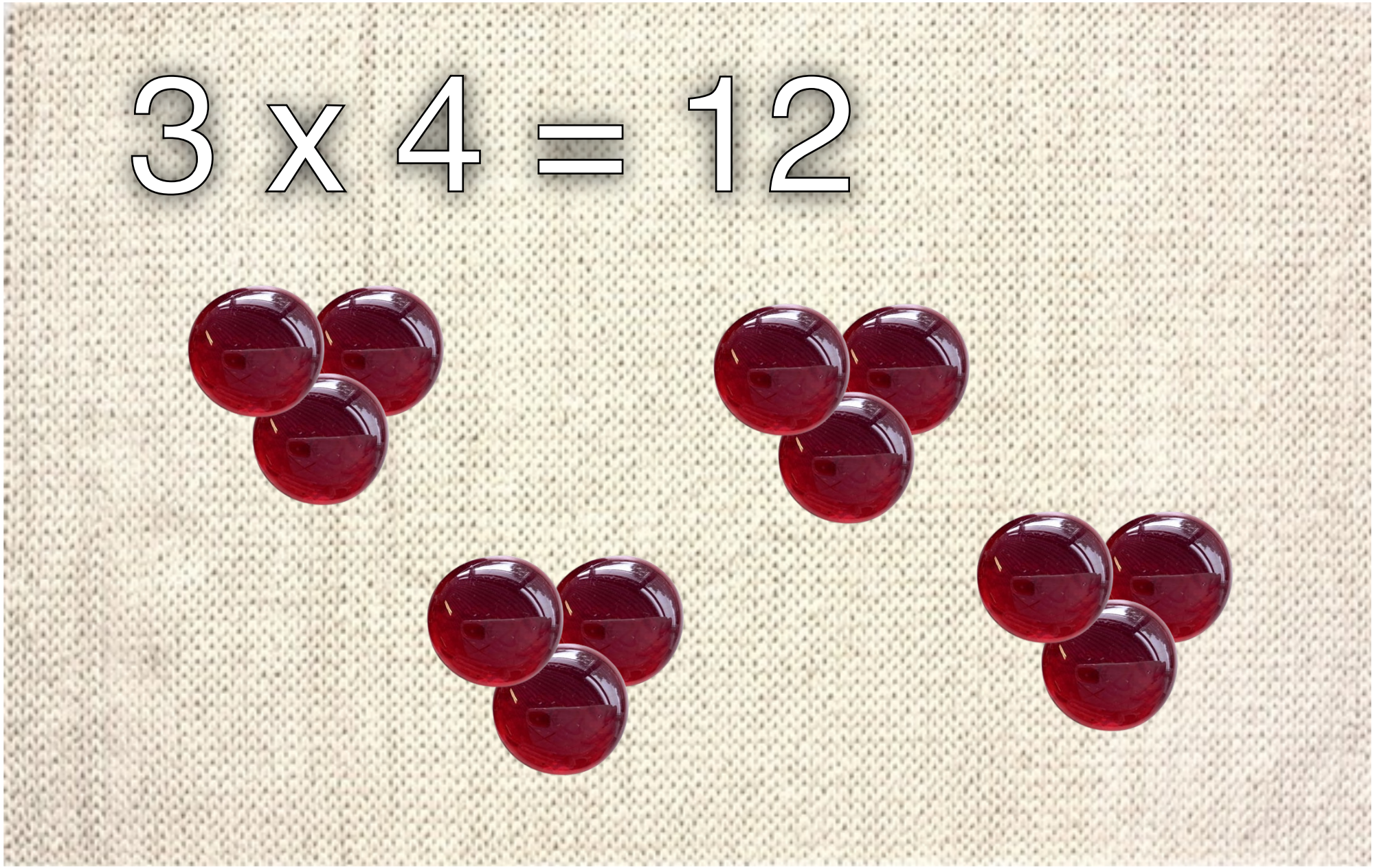
Handout 1

Familiar Models

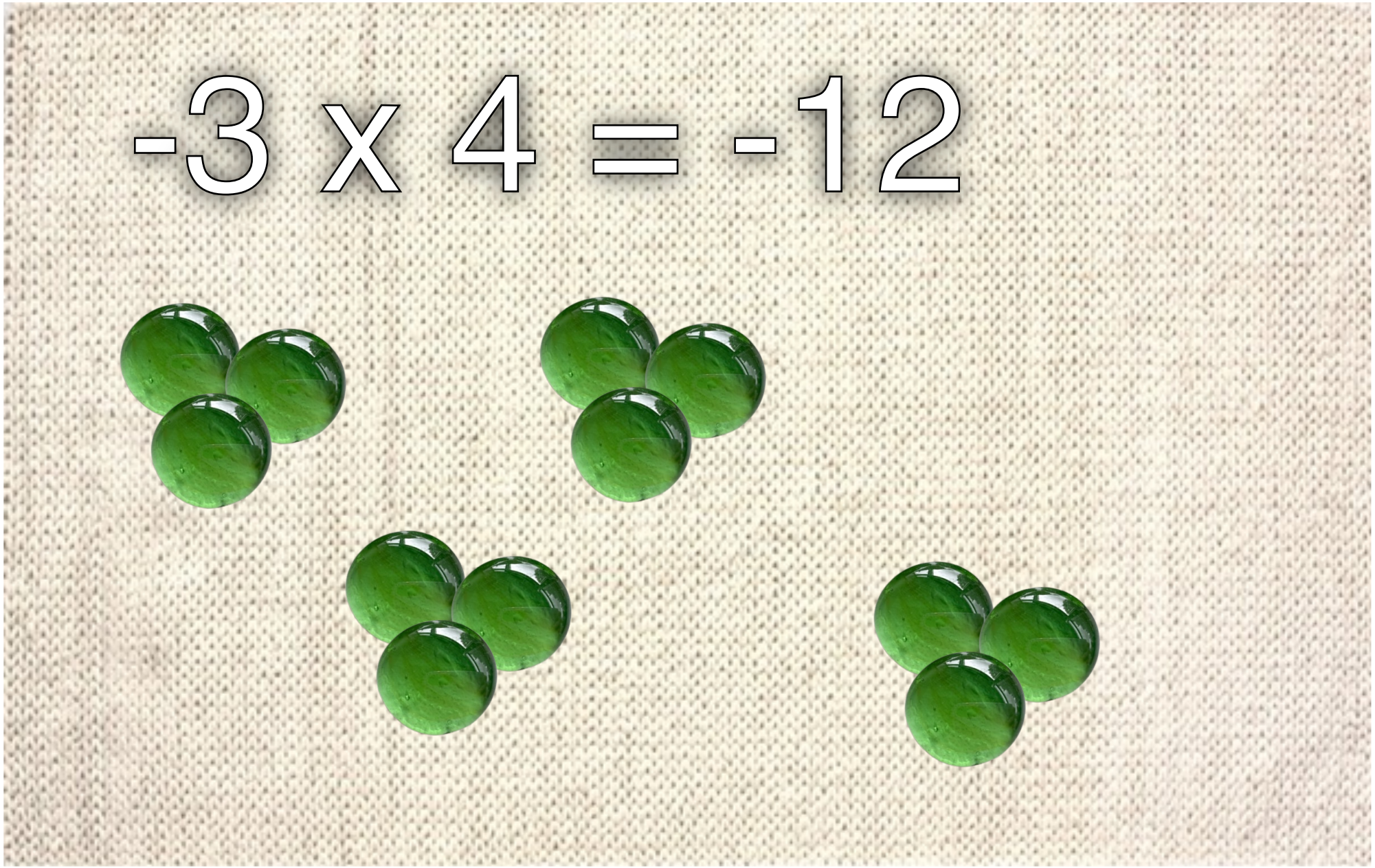
- walking on a number line
- bags of weight in a hot air balloon

Potential Models

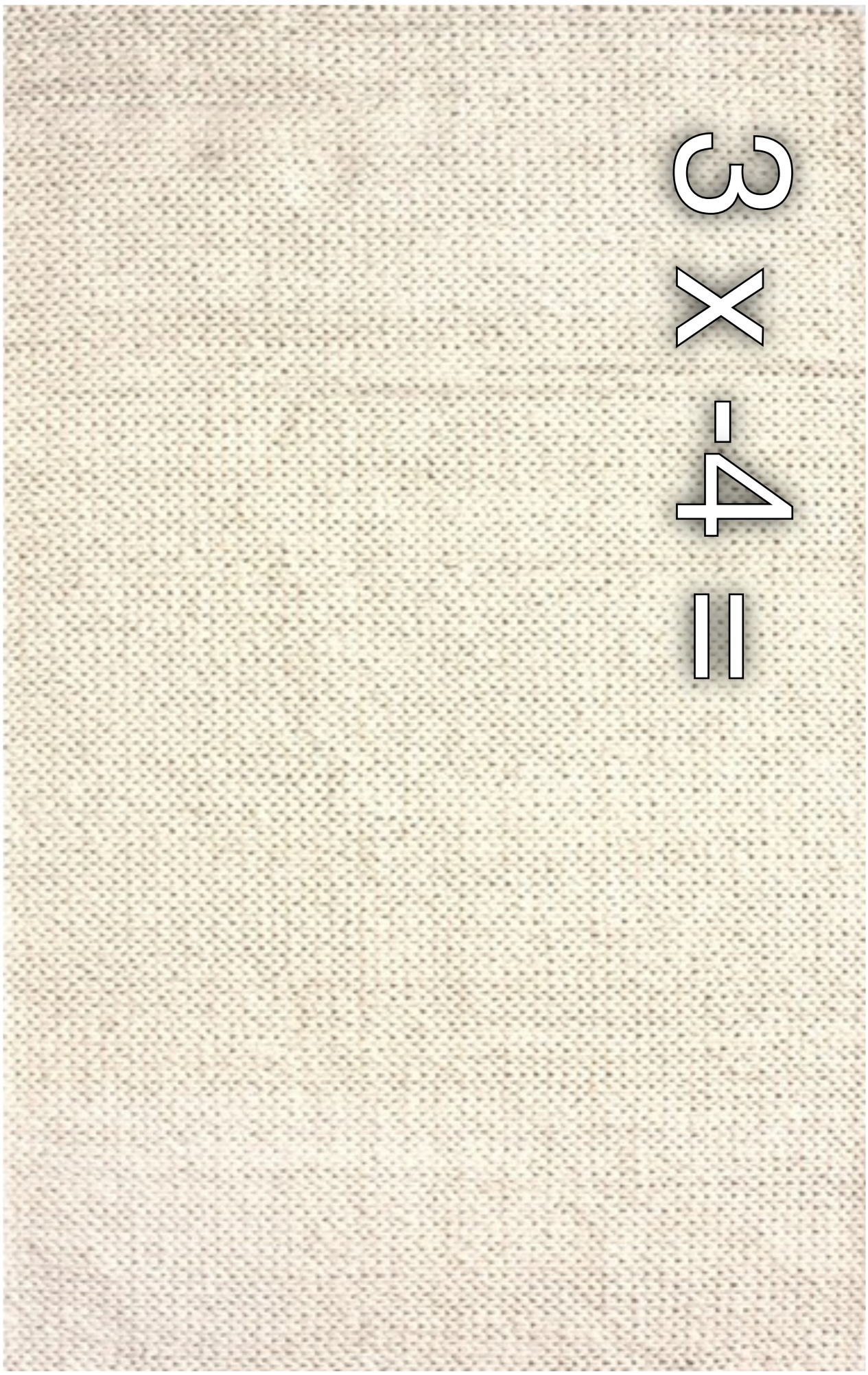


Potential Models

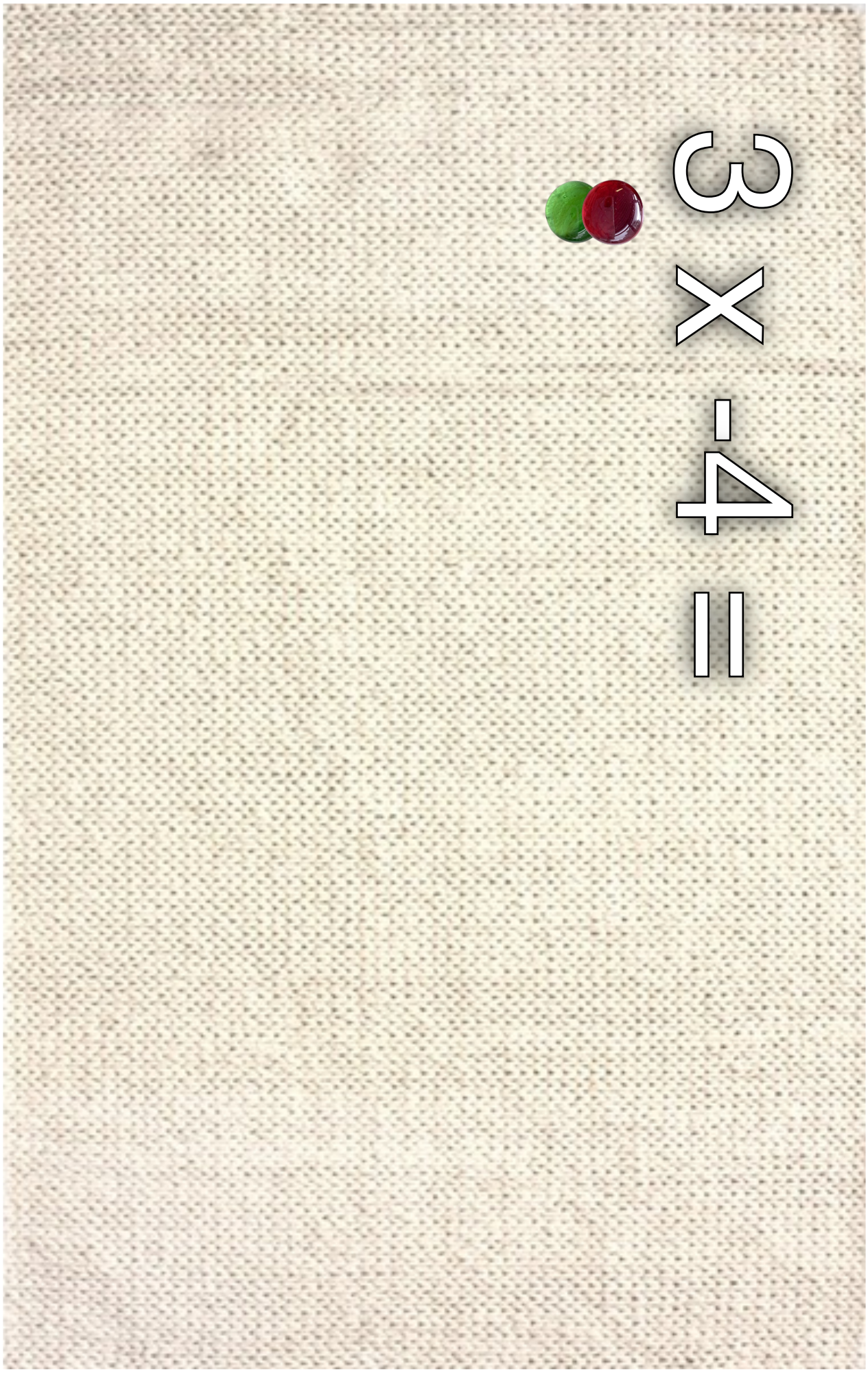
$$-3 \times 4 = -12$$



Potential Models

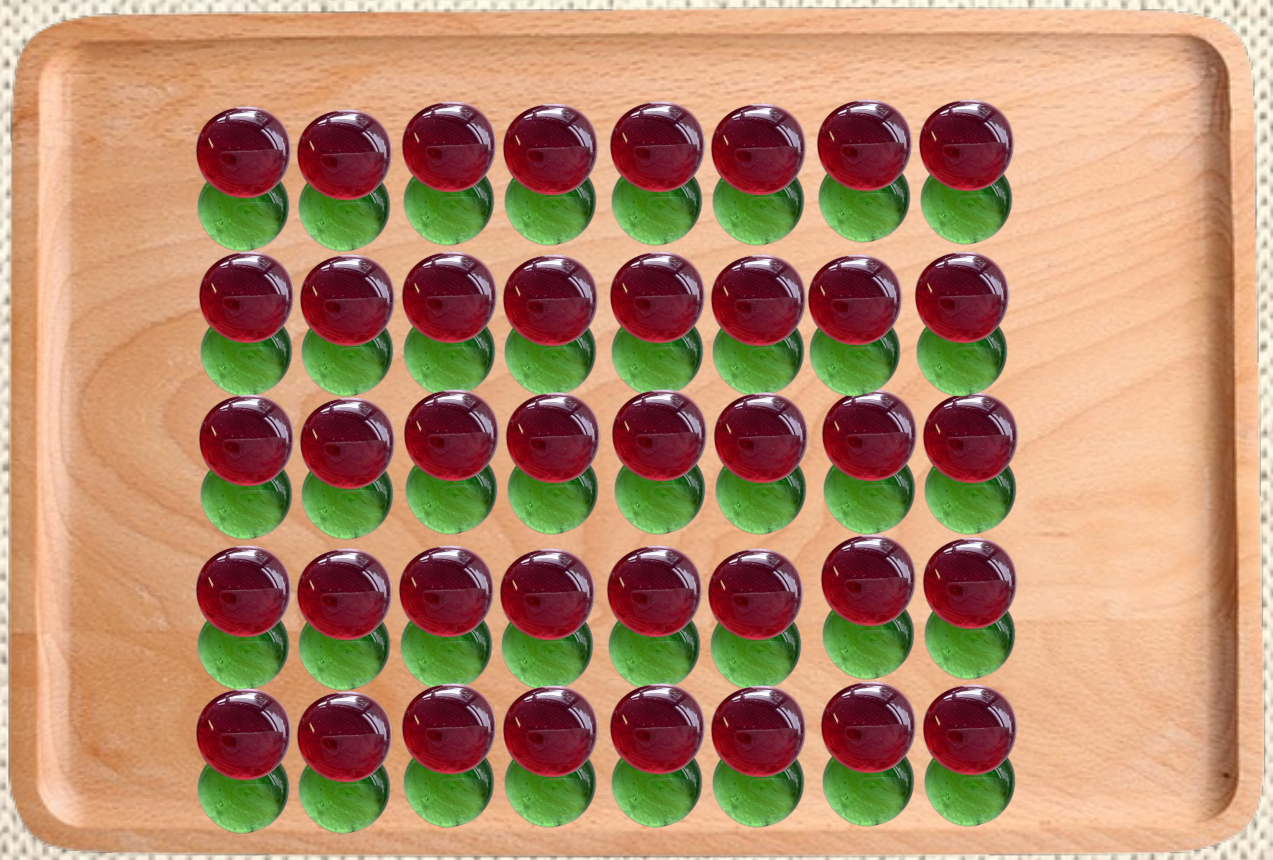


Potential Models

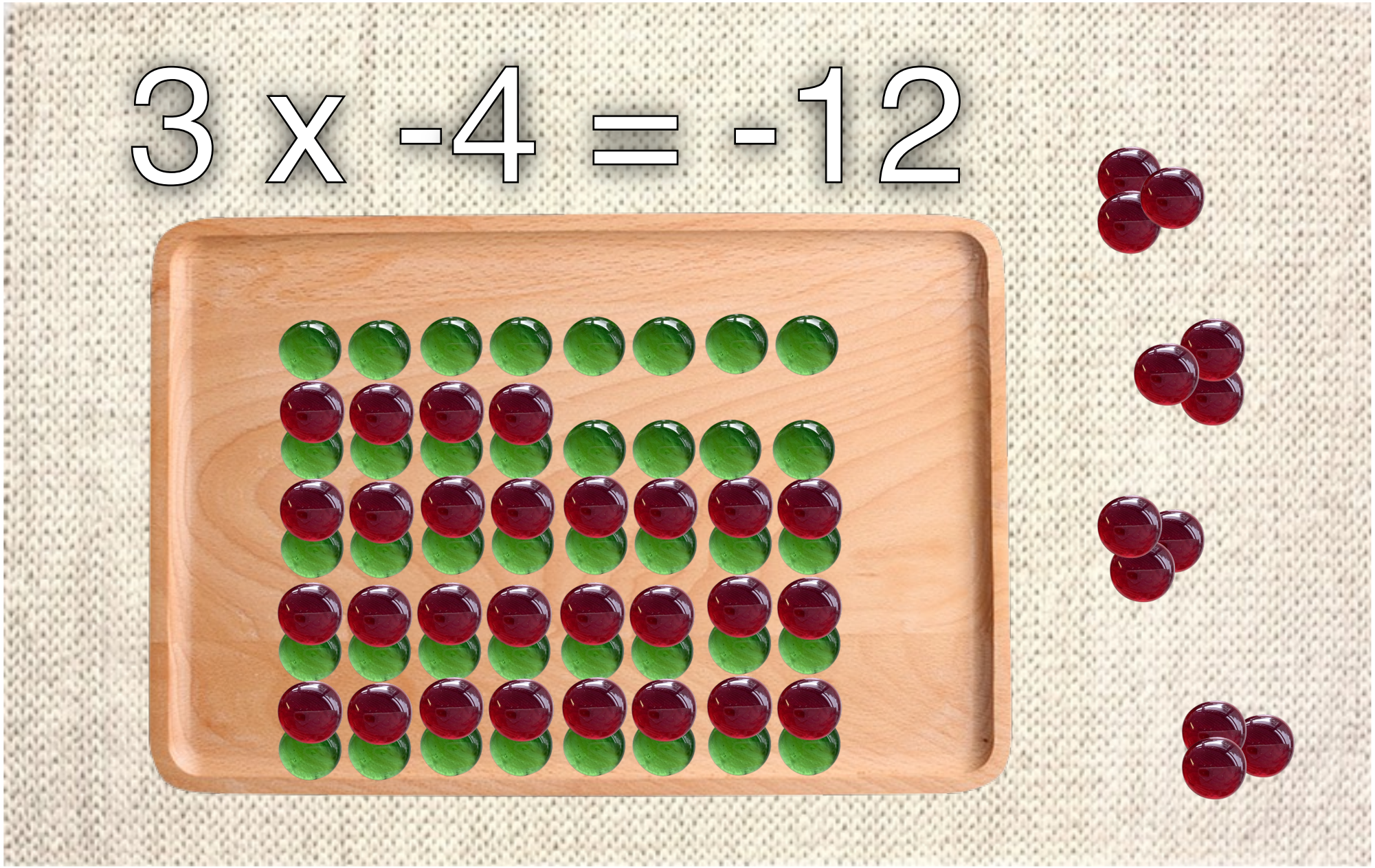


Potential Models

$3 \times -4 =$

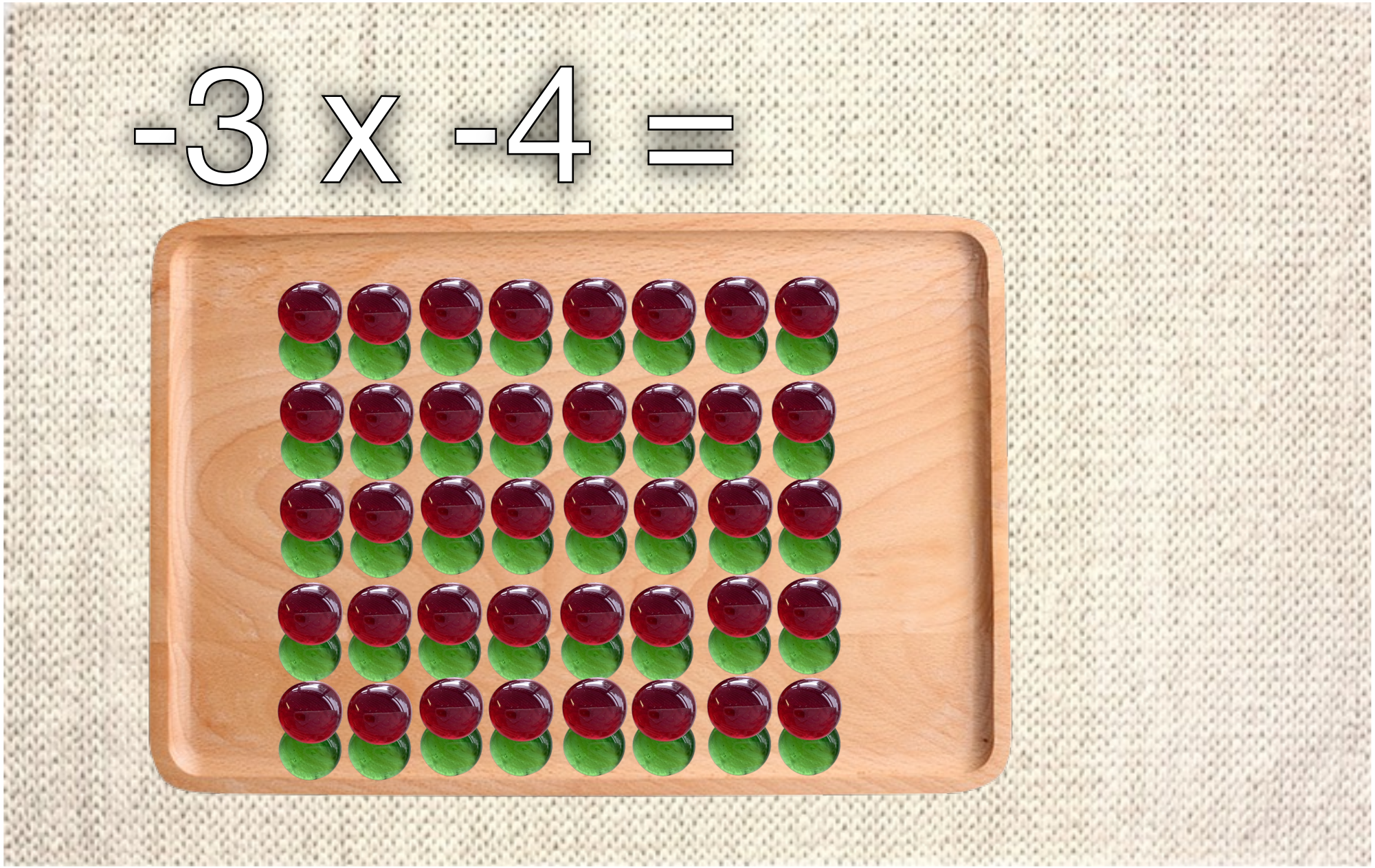


Potential Models

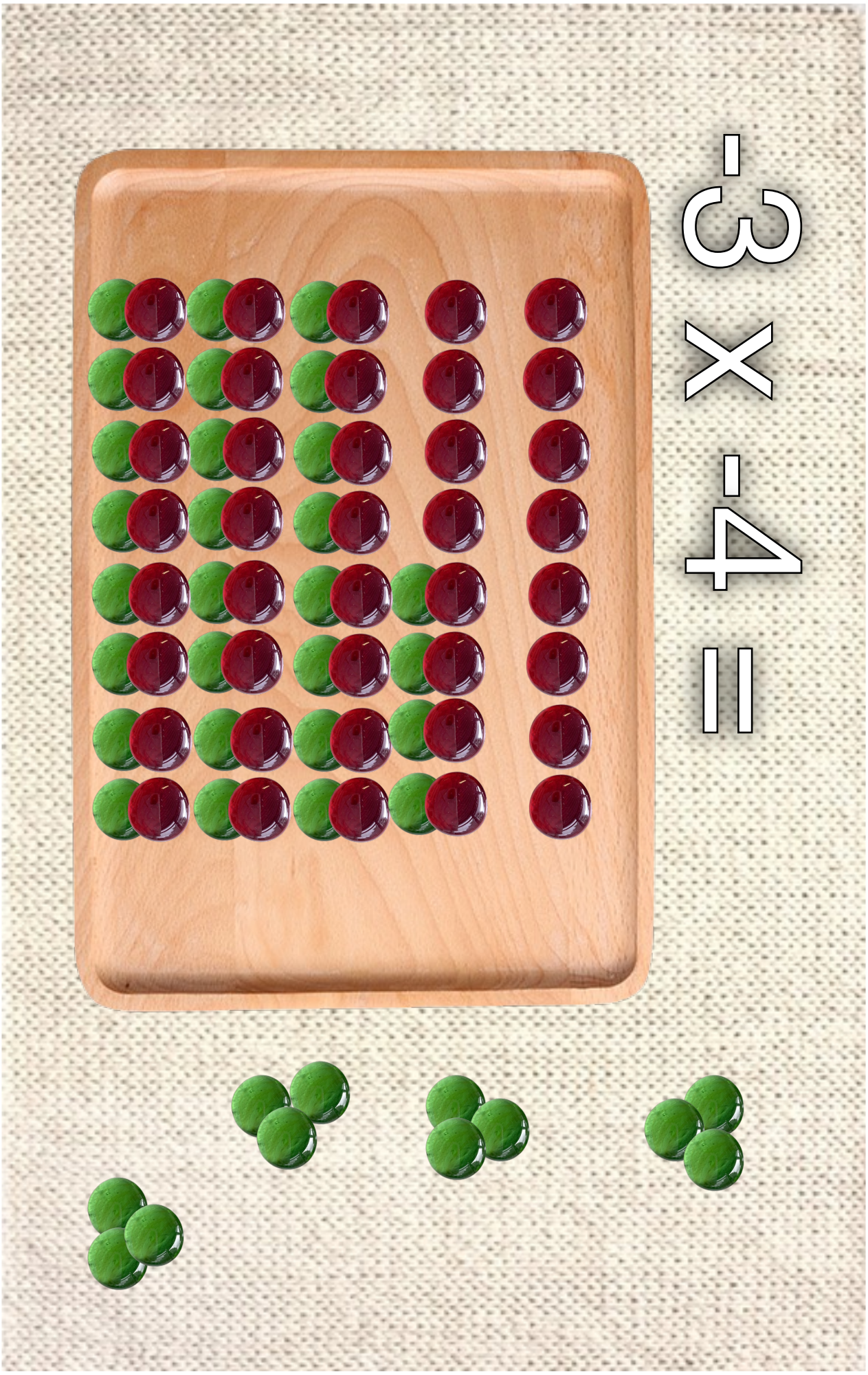


Potential Models

-3 x -4 =



Potential Models



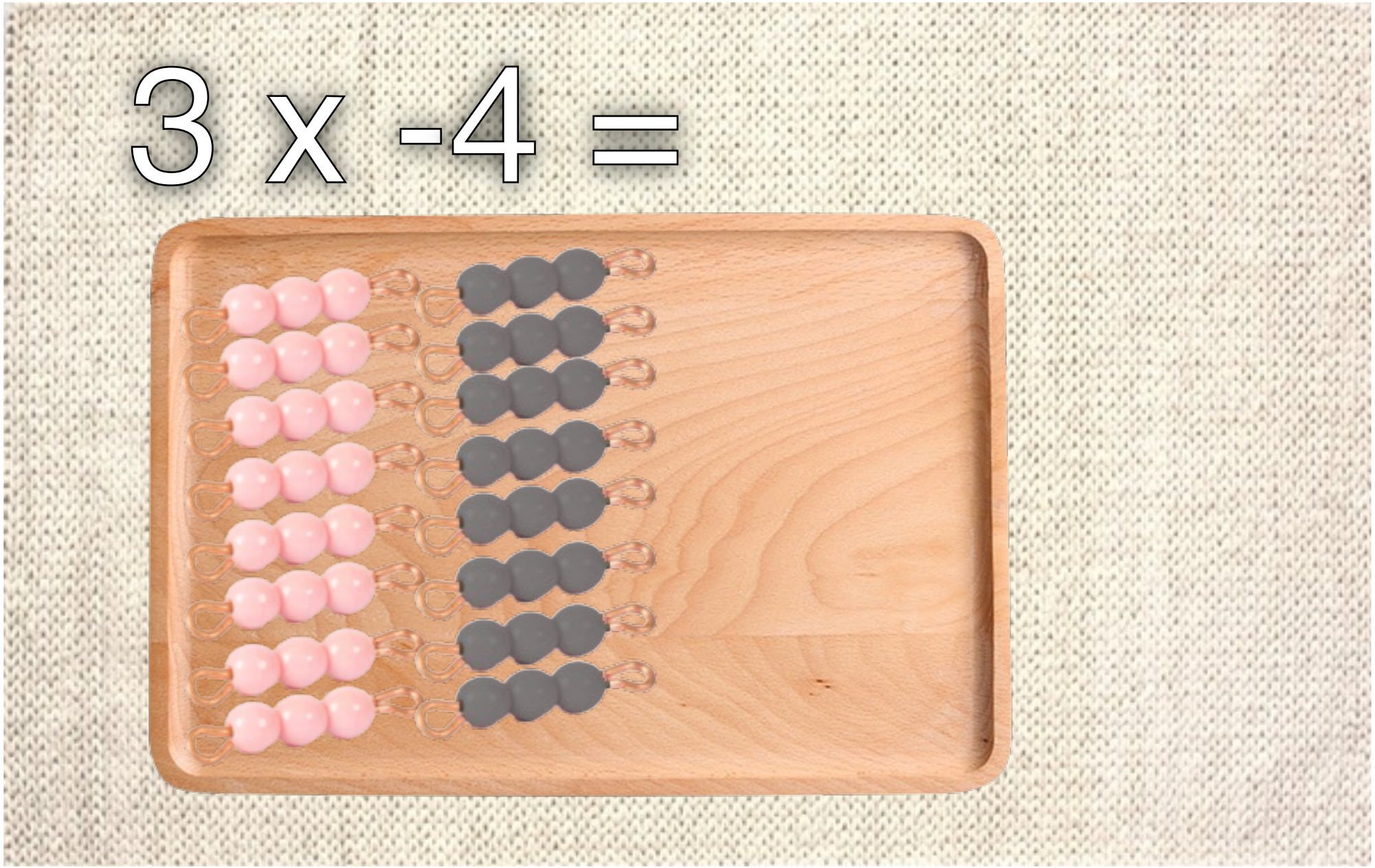
Potential Models

$3 \times 4 = 12$

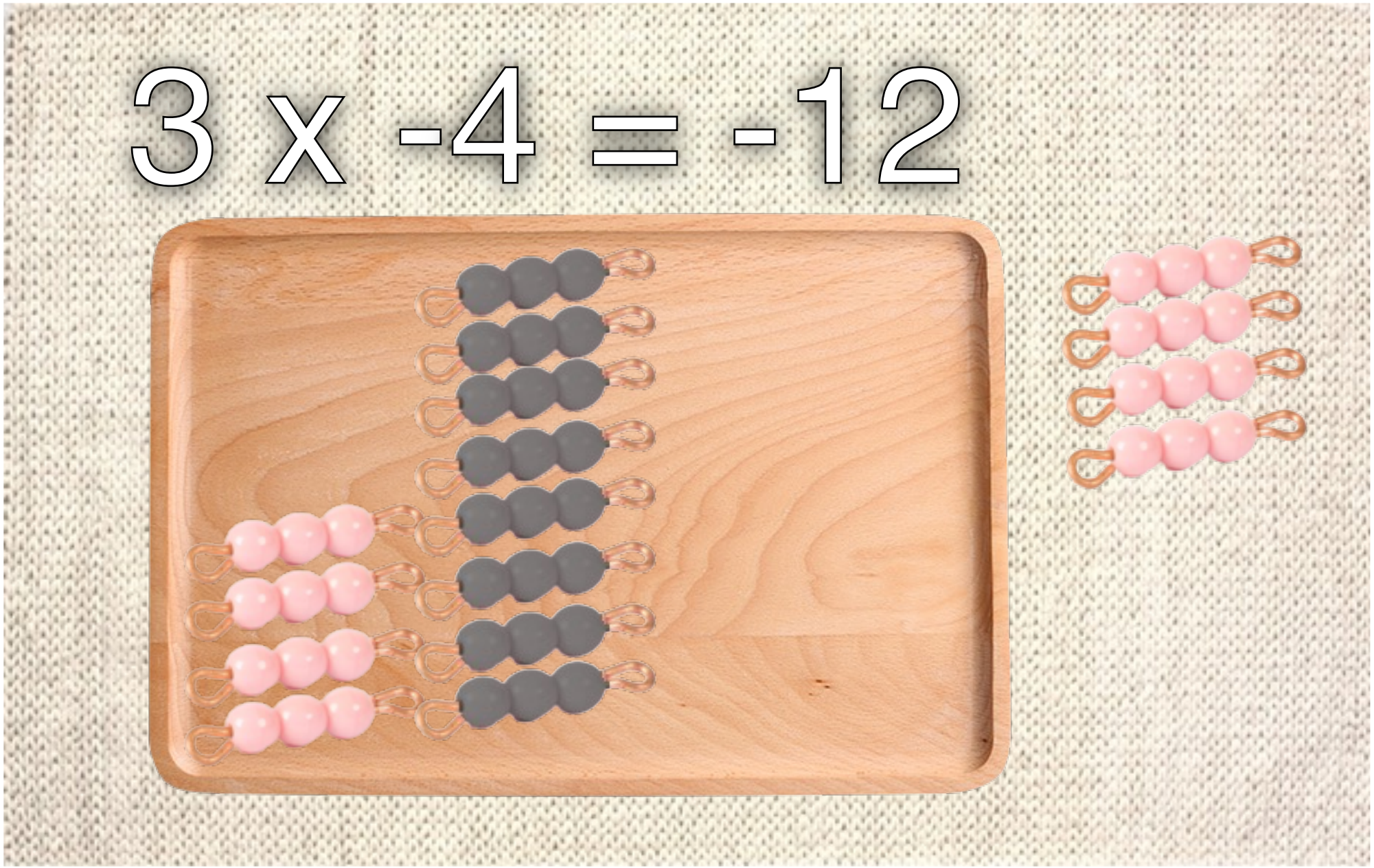
$-3 \times 4 = -12$

Potential Models

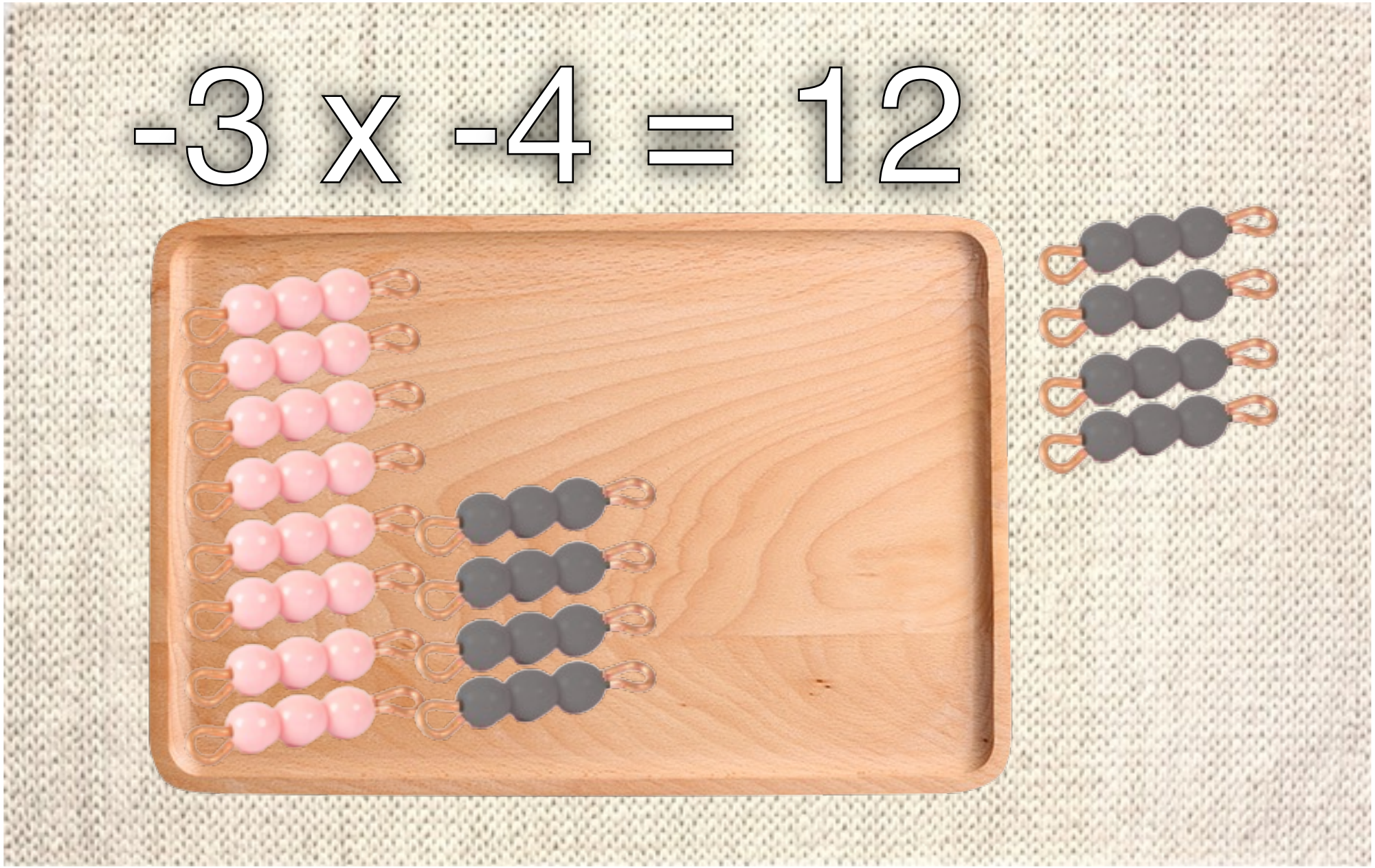
$3 \times -4 =$



Potential Models



Potential Models



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Task Analysis

3 minutes

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- How did this task guide you to **reason** *algebraically*?
- What **student actions** made the experience meaningful?
- What **teacher actions** made the experience meaningful?

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1

Observe patterns and *share* what you **notice**.

Student Actions…

> “When I multiply two negative integers, I keep getting a positive integer for the answer.”

Montessori Curriculum

Montessori Curriculum

Teacher Actions

- Create strategic problem sets.
- Use questioning.
- Ask students to consider an equation without solving.

Observe Without *Solving*

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"What do you know about the answer to $-4 \cdot -6$ without solving?"

Standards for *Mathematical Practice*

- **MP7**: Look for and make use of ***structure***.
- **MP8**: Look for and express regularity in *repeated reasoning*.

2

Make a
general *claim* or
conjecture.

Make a Conjecture

> A **conjecture** is a statement that has been **proposed** **as true** and appears to be correct but has ***not yet been proven or disproven***.

Connecting Arithmetic to Algebra
Susan Jo Russel · Deborah Schifter · Virginia Bastable

Sounds Like…

"Do you think that is always true? How can we explain this rule?"

Teacher Actions

- Teach kids explicitly about conjectures.
- Practice making conjectures out loud and in writing.
- Refine language by offering tier 3 vocabulary.

Shifting Class *Culture*

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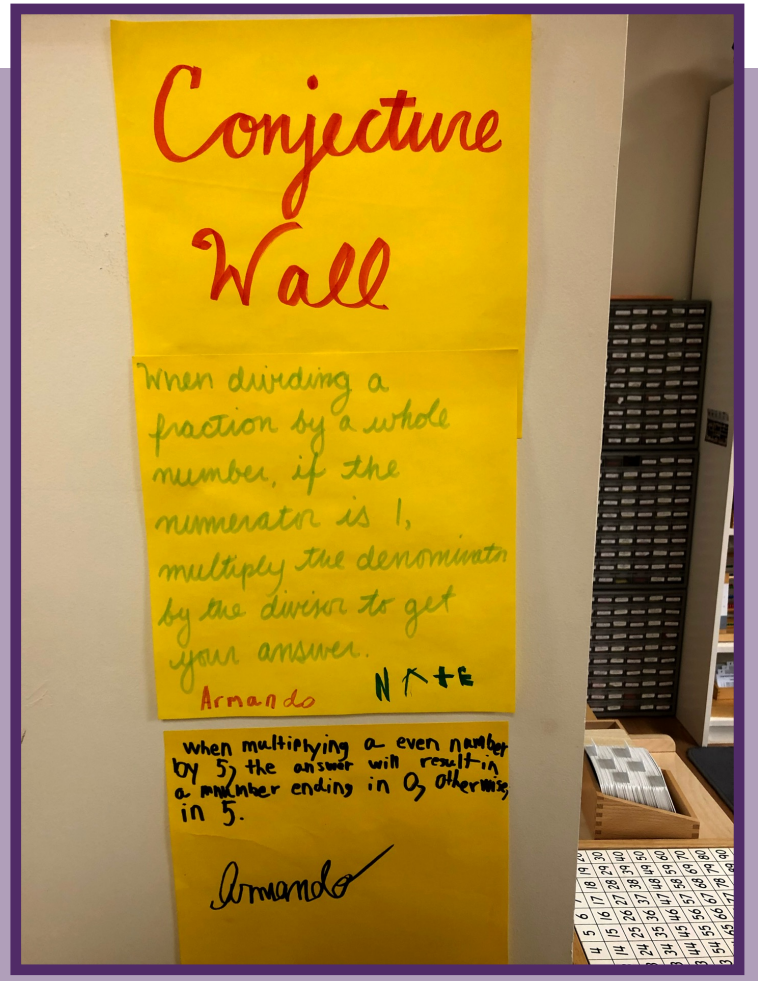
- Create a conjecture wall
- Allow students to revise their conjectures over time
- Celebrate conjectures

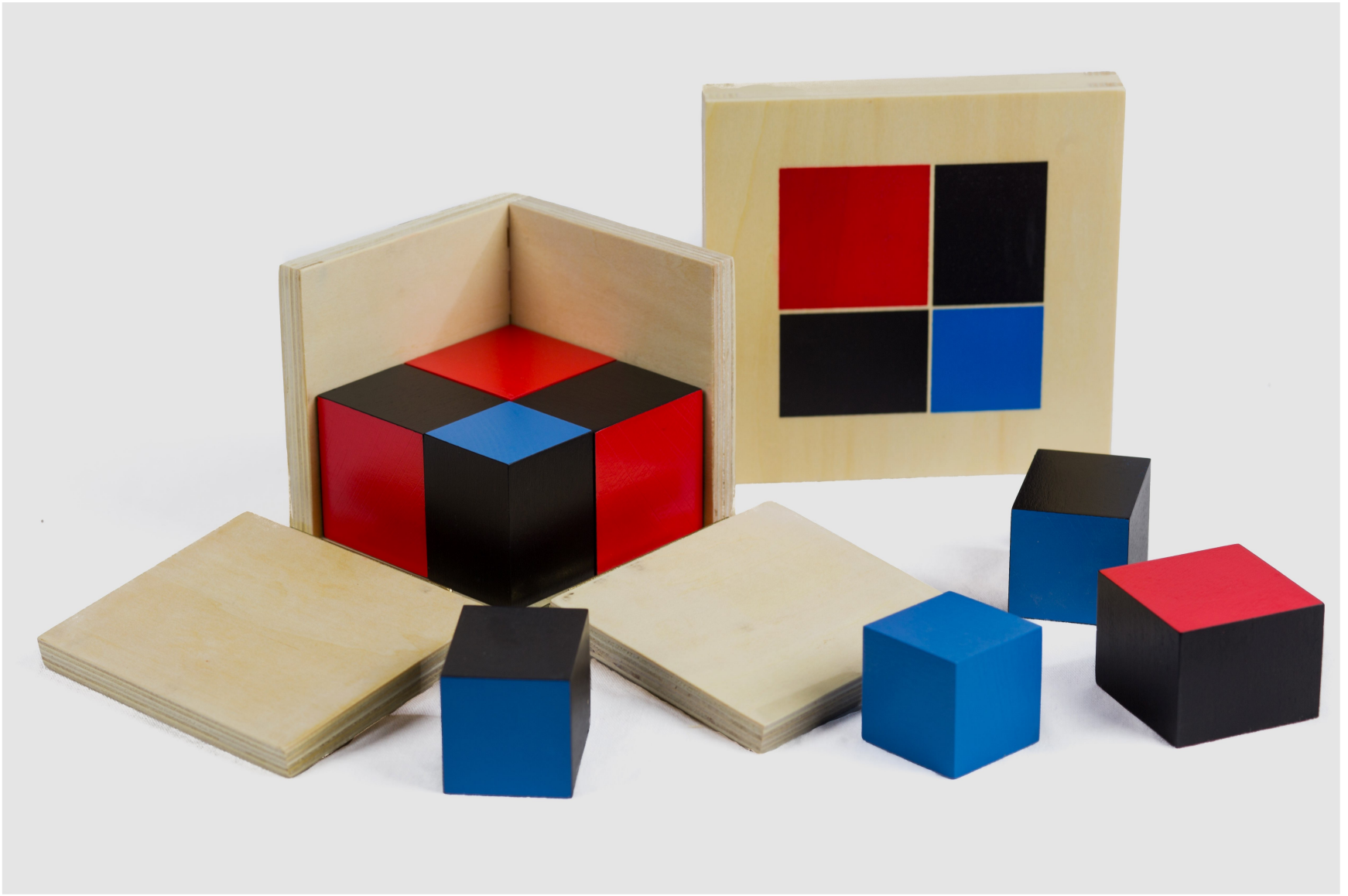


3

Develop a **representation** that **proves** the claim is *always true*.

Montessori Curriculum

Montessori Curriculum



4

Represent the claim using **algebraic notation**.

Looks Like…

$$a \cdot b = (-a) \cdot (-b)$$

5

Consider how the *claim changes* for **different domains** of numbers.

Sample Conjecture

When you multiply two factors, the product is larger than the factors.

Sample Conjecture

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When you multiply two **positive** factors, the product is larger than the factors.

Sample Conjecture

When you multiply two factors, the **absolute value** of the product is larger than the factors.

Sample Conjecture

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When you multiply two factors **greater than 1**, the product is larger than the factors.

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Lesson Planning

- Synthesize and apply these practices to your own context

5 minutes

Reasoning Algebraically

Lesson Planning

Lesson Concept: _____

1. Observe patterns and share what you notice.

What **patterns** do you hope children will observe?	
Are there particular **materials** that would best illuminate these patterns?	
Create a set or sets of **equations** that will help illuminate the pattern.	
What **questions** can you ask to guide children towards observing the pattern?	
Ask students to consider an equation **without solving** it.	

2. Make a general claim or conjecture. What claim do you hope children investigate?

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Handout 2

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